RBC and Loss of Skills During Unemployment

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Abstract

In this paper we propose a RBC model of frictional labor markets with two types of workers: high-skill and low-skill workers, where high-skill workers may suffer from a depreciation of their human capital while unemployed.

We estimate the parameters of the model via maximum likelihood and analyze the cyclical properties of the model. We also contribute to the literature that tries to explain the different performance of European and US unemployment conciling the macro and micro evidence

Keywords: skills, turbulence, unemployment, unemployment benefit matching

JEL-Codes: J,

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1 Introduction

It is well known in the literature of unemployment that until the second half of the seventies, the European unemployment was significantly lower than the American unemployment, and that since the late seventies and during the eighties the tendency changed and the European unemployment started to steadily rise while the American unemployment continue to fluctuate around its post-World War II value.

The increase in European unemployment was largely caused by a lengthening of the average duration of unemployment spells. So although many Europeans leave unemployment relatively quickly, a significant fraction of workers become trapped in long-term unemployment and have little chance of finding the jobs they want.

The table below is intended to present evidence on the statements above. In fact, we observe that whereas unemployment in US remain quite stable over time along the period between 1970-2000, the figures have more than doubled for continental Europe. Moreover, a larger fraction of long-term unemployment has accompanied this tendency reaching, on average, values close to 70 per cent of total unemployment in 1995.

Nowadays, many European countries still suffer from a chronic unemployment rate. In big economies such as Germany, France and Spain, around 8 per cent of the labor force is unemployed and a high percentage of them is classified as long term unemployed. Therefore, it seems necessary, given the large fraction of labor force that might be affected by this problem to properly understand the sources of this problem and the creation of a suitable framework where to analyze it.

Table 1: Unemployment and long-term unemployment

Country	Unemployment			Long term unemployment		
	1974-1979	1980-1989	1990-1999	1974-1979	1980-1989	1990-1999
Belgium	4.0	9.54	8.47	74.9	87.5	77.7
France	4.5	9.0	10.60	55.1	63.7	68.9
Germany	3.2	5.9	7.49	39.9	66.7	65.4
Netherlands	3.80	8.16	5.41	49.3	66.1	74.4
Spain	5.2	17.5	22.9	51.6	72.7	72.2
Sweden	1.9	2.5	7.21	19.6	18.4	35.2
U. Kingdom	5.0	10.0	8.00	39.7	57.2	60.7
USA	6.7	7.2	5.71	8.8	9.9	17.3

Source: *Ljungqvist and Sargent* (2002) *and AMECO European Commision*

Related to the determinants of the persistence in unemployment, Pissarides (1992) showed that when unemployed workers lose some of their skills, the effects of a negative temporary shock to employment can persist for a long time. The key mechanism that drives the result is a variant of the "thin market externality" that reduces the demand of jobs when duration of unemployment increases. A similar underlying idea we find in Blanchard and Diamond (1994) who study the relationship between "ranking" -or the preference of employers for short-term unemployed workers- wages and unemployment. The hypothesis of loss of skills during unemployment has also been used in the literature to explain the differences between unemployment rates in Europe and US. Ljungqvist and Sargent (1998) is the first paper that introduces this "turbulence" shock in the literature.

Given that none of the papers above study the cyclical behavior of unemployment and other macro-variables, it seems sensible, once we have understood which are the key problems of labor markets nowadays (i.e. the steadily increase of unemployment since the late 70s and the large fraction of long-term unemployed), to try to embed them into a standard real business cycle model so as to construct a suitable framework for policy making. This is a quite ambitious target and the model presented here ties to contribute to this literature.

Our starting point would be the seminal papers that introduce frictional labor markets into a RBC framework (Merz (1995) and Andolfatto (1996)). These two papers outperform previous studies in terms of explaining the performance of the macroeconomic variables along the business cycle. However, as Hall (2005) and Shimer (2005) pointed out, there is still room for improvement, mainly in terms of volatility and persistence of vacancies and unemployment, and therefore of the labor market tightness. Shimer suggests that this deficiency could be overcome by introducing sticky wages. We will analyze, as well, how the assumption introduced in this model, i.e., the loss of skills, can contribute or not to better understand the propagation mechanism of unemployment, and consecutively, of labor market tightness.

Therefore, with this paper we want to contribute to the literature of unemployment paying special attention to long-term unemployment. In particular, we study the worker's depreciation of human capital during long spells of unemployment and its implications for the business cycle and the persistence of unemployment. We propose a DSGE model in which the labor market is explicitly modellized as a frictional market and allow for the depreciation of human capital during long spells of unemployment. We bring the model to US data using maximum likelihood estimation which allows us to evaluate the model and extract interesting conclusions related to literature analyzing the rise of European unemployment.

Our model seems to fit the data quite well, increases the persistence of unemployment and reconciliates the micro and macro explanations given by the literature to understand the different behavior between the European and the American unemployment. In the next section, we review in detail this literature.

The structure of the paper is the following: in section 2, we review the explanations that the literature has given to the different performance of unemployment between Europe and US. In section 3 we present the model and we estimate it via maximum likelihood in section 4. Section 5 contains the results, and we conclude in section 6.

2 Explanations to the European unemployment

As we have seen, starting in the late 1970s and continuing through the 1980s unemployment steadily increased in Europe and became the main economic issue facing Europe. The first attempts to explain this increase in unemployment relied on the role played by labor market institutions such as employment protection legislation, both the duration and generosity of unemployment insurances (see Martin, 1996) and the role of firing costs (see Bentolila and Bertola, 1990). The problem with this explanation is that also during the sixties and seventies, when the unemployment in Europe was lower than in the US those labor market institutions existed already (see Krugman, 1987).

Another early attempt to explain this rise in unemployment focused on the negative effect that some macro-shocks could have had on unemployment. Among this macro shocks we find the oil-price shock of 1973 and 1979, the TFP growth slowdown since the early 1970s and other shifts in labor demand experienced since the 1980s. This interpretation was also challenged by Phelphs (1994) who saw improbable that these initial shocks, which indeed have been largely reversed lately, could still be responsible for high unemployment more than fifteen years later. Phelps, for example, emphasized factors that increased the real interest rate and consequently the rate of unemployment.

The stability of European labor market institutions before and after the late seventies and the difficulty of aggregate shocks to explain the persistence of unemployment, lead to another stream of explanations that consider the possibility that changes in the economic environment, in particular aggregate macroeconomic shocks, interacted with labor market institutions to unleash persistently high unemployment. This hypothesis blamed adverse shocks for the initial increase in the rate of unemployment, and labor market institutions for the persistence of this rate.

The explanation based on the interaction of adverse shocks with adverse labor market institutions has been studied in detail by Blanchard and Wolfers (1999). They call the attention about the potential to explain not only the increase in unemployment over time through adverse shocks and the fact that some institutions may affect its persistence but they can also explain cross country differences[1]. In a companion paper Blanchard and Wolfers, (2000) look, through panel data specifications, at the empirical evidence about the role of macro shocks, the role of institutions and the role of the interaction between shocks and institutions in accounting for the European unemployment. Their results suggest that specifications that allow for shocks, institutions and interactions can account both for much of the rise and much of the heterogeneity in the evolution of unemployment in Europe.

The second big stream of explanations given to the high European unemployment focus on the interaction of micro-shocks and labor market institutions rather than focussing on the interactions of those institutions and aggregate shocks. The two main interpretations of these findings come from Bertola and Ichino(1995) and Ljungqvist and Sargent (1998). Bertola and Ichino show that given the rigid wages and the high firing costs that prevail in Europe during the 80s, a higher likelihood of negative shocks in the near future decreases labor demand by hiring firms. And as long as the wage rate does not fall, the equilibrium unemployment rate would

[1] Recently, Nickell et. al. (2005) consider a plausible story the fact that in response to the initial increase in unemployment, governments reacted by taking the wrong measures. They explain how governments in order to alleviate the pain of unemployment increased the generosity and duration of unemployment or in order to limit the increase in unemployment, they tried to prevent firms from laying off workers through tougher employment protection or even . To better share the burden of low employment, they used early retirements and work sharing to better share the burden of low employment. All these measures then in turn increased unemployment even as the initial shocks disappeared.

rise. This explanation remind us, the "thin market externality" reasoning proposed by Pissarides (1992).

Ljungqvist and Sargent's series of papers, LS from now onwards, advocate for the interaction of shocks to individual worker's human capital, *turbulence* in their words, and generous unemployment benefits to produce long-term unemployment in Europe. In particular, they assume that in the late 70s and during the 80s, the probability of suffering from a depreciation of human capital increased and unleash the following mechanism: Imagine a worker who suddenly loss his job. Once unemployed he receives an unemployment benefit proportional to his former wage and become a low-skill worker. If any, he is going to receive job offers corresponding to this low-skill level and accordingly, he is going to be offered low wages. It easily could be that those low-wages do not cover the reservation wage of the worker which we can identify with the high-skill unemployment insurance. If this is the case, he is going to reject the offer and will become trapped in unemployment.

More recently, a new hypothesis come up to the fore. Prescott (2004), advocates for the role of tax rates, in particular the effective marginal tax rate on labor income, in accounting for the changes in the relative labor supply across time and across countries. Interesting findings of this study are that when European and US tax rates were comparable, European and US labor supplies were comparable and that the low labor supplies of Germany, France and Italy during the nineties are largely due to high tax rates.

So, nowadays we rely on at least three possible potential explanations for the high European unemployment: the combination of aggregate macro shocks and labor market institutions, the combination of micro-shocks and labor market institutions and the impact of the evolution of labor taxation on labor supply. But to disentangle the exact effect of labor market institutions on unemployment is still an issue we need to resolve before we can declare, in Blanchard's words, an intellectual victory.

As we will see our results, conciliate the micro and macro-shock based explanations in the following way: according to our model, what we find is that is a macro shock that generates the initial increase in the pool of unemployment but that is the micro shock in combination with generous unemployment benefits what contributes to its persistence.

3 The model

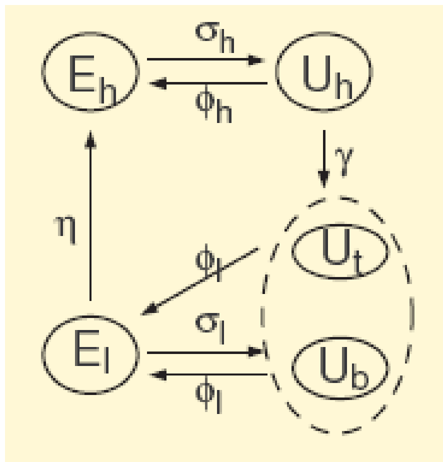
The economy is populated by a continuum of infinitely lived agents whose sum is normalized to one. Workers are assumed to be either high-skill,h, or low-skill,l. High skilled workers who have just lost their jobs retain their skill for a certain period of time. The loss of skill occurs over time and is modeled as a random process following a Poisson distribution with parameter γ, i.e. with probability γ a high-skill unemployed worker is going to suffer a depreciation of his human capital and be transformed into a low-skill unemployed. On the other hand, there is a probability η that a low skill worker upgrades his human capital and become a high skill worker.

Both types of workers can exogenously lose their jobs with probabilities σ_h for the high skill and σ_l for the low-skill. The probabilities of leaving unemployment or equivalently, the probabilities of finding a job are ϕ_h for the high-skill and ϕ_l for the low-skill.

Unemployed workers receive an unemployment insurance when losing their jobs which is a fraction of the wage they had while working. In this way we have three types of unemployed workers: U_h represent the pool of unemployed workers with high skills and high unemployment benefit, U_t represent the pool of unemployed workers who have suffered a depreciation of their human capital but still receive high unemployment benefits and U_b represents the pool of unemployed workers with low skills and low unemployment benefits.

When looking for a job, we pool the low-skill searchers with low and high unemployment benefits, and create a group $U_l = U_t + U_b$. Firms opening vacancies for workers with low-skills face the supply U_l.

Therefore, our model can be summarized through the flows represented in the next figure.



Flows of workers

Firms and Technology - The production sector is made up of a large number of identical competitive firms. The production technology is represented by a constant returns to scale Cobb-Douglas production function. Therefore, there exists a representative firm which uses capital, K_t, and labor, T_t, to produce the aggregate good, Y_t, according to the following technology

$$Y_t = \xi A_t K_t^{\theta} (\ell^t T_t)^{1-\theta}$$

where $\ell > 1$ measures the gross rate of labor-augmenting technological progress. The fact that we have a deterministic growth rate, would make necessary to detrend the variables in such a way that in equilibrium the economy would converge to a steady state in which the detrended variables of the model would remain constant. We define the detrended variables, which will be represented in small letters, as: $y_t = Y_t/\ell^t$, $k_t = K_t//\ell^t$, $t_t = T_t$, $a_t = A_t$, $u_{j,t} = U_{j,t}$, $n_{j,t} = N_{j,t}$, $h_{j,t} = H_{j,t}$, $w_{j,t} = \ell^t w_{j,t}/\ell^t$, $v_{j,t} = V_{j,t}$, $a_{j,t} = \ell^t a_{j,t}/\ell^t$, $m_{j,t} = M_{j,t}$, $i_{j,t} = I_{j,t}/\ell^t$. In what follows we will work with the stationary model. This means that the production function above can be stated as:

$$y_t = \xi a_t k_t^{\theta} (t_t)^{1-\theta}$$

The TFP shock a_t follows a first order autorregresive process

$$\ln(a_t) = \rho \ln(a_{t-1}) + (1-\rho)\ln(\overline{a}) + \varepsilon_t^a$$

where $a^* > 0$ represents the steady state value, and $-1 < \rho < 1$. The serially uncorrelated innovation ε_t^a is assumed to be normally and independently distributed over time with mean 0 and variance σ_ε.

The firm hires capital, k_t, and labor, t_t and opens vacancies for high and low-skilled workers, v_h and v_l, to maximize the expected present value of cash flows,

$$\sum_{t=0}^{\infty} \Delta^t \left[F(a_t, k_t, t_t) - w_{h,t} n_{h,t} h_{h,t} - w_{l,t} n_{l,t} h_{l,t} - r_t k_t - a(v_s + v_l) \right]$$

subject to the laws of motion of employment (1) and (2) specified below. a denotes de cost of opening a vacant v_j, with $j = h, l$ and Δ^t is the discount factor for the firm, with $\Delta^t = \frac{\beta Uc(s')}{Uc(s)}$. The amount of labor included in the production function is defined in efficiency units as follows

$$t_t = n_{h,t} h_{h,t} + \tau n_{l,t} h_{l,t}$$

where $n_{j,t}$ denotes the number of workers of type j in period t and $h_{j,t}$ denotes the number of hours worked by each type of worker. $\tau < 1$.

Since the labor market is frictional, the laws of motion for the two types of workers (high and low skill) are defined as:

$$n_{h,t+1} = n_{h,t}(1-\sigma_s) + q_{h,t}v_{h,t} + \eta n_{l,t} \tag{1}$$

$$n_{l,t+1} = n_{l,t}(1-\sigma_l) + q_{l,t}v_{l,t} - \eta n_{l,t} \tag{2}$$

where $q_{j,t}$ represents the perceived probability that a vacancy of type j get matched with an unemployed worker of the same type. $\eta n_{l,t}$ represents the fraction of low-skilled unemployed workers that suffer an upgrade of skills every period. This movement occurs with exogenous probability η. Thus, upgrading follows a Poisson process with η rate which is independent of other processes in the model.

The labor market - The labor market is modellized as a frictional market in which firms and workers engage in employment relationships. The total number of matches per unit of time is represented by the following technology

$$m_{j,t} = m(v_{j,t}\ ,\ u_{j,t})$$

where $u_{j,t}$ represents the total number of type j searchers and $v_{j,t}$ the total number of vacancies of type j. This matching function is increasing in both arguments, concave and homogeneous of degree one.

The job vacancies and the unemployed workers that are matched at any point in time t, are randomly selected from the sets v and u. Therefore, the process that changes the state of vacant jobs to filled vacant is a Poisson with rate $q_{j,t}=\frac{m_{j,t}}{v_{j,t}}$. Similarly, unemployed workers move into employment with probability $\phi_{j,t}=\frac{m_{j,t}}{u_{j,t}}$.

The empirical literature has further found that a log-linear Cobb-Douglas approximation of the matching function fits the data well[2]. So, in our model, the total number of matches at time t is given by a Cobb-Douglas matching function in the total number of searchers and vacancies of type j.

[2]see Pissarides (1990), ch.1 and Petrongolo and Pissarides (2001) for a survey.

$$m_{j,t} = \chi_j v_{j,t}^{\alpha j} u_{j,t}^{1-\alpha_j}$$

χ_j is called the "efficiency parameter" of the matching function. Under the Cobb-Douglas specification above, the probability of finding a job, ϕ_j increases with the tightness ratio $(\frac{v}{u})$ with elasticity less $1 - \alpha_j < 1$.

Households and Preferences - The economy is populated by identical, infinitely-lived households. In each household there are high skilled and low skilled workers. The measure of type j workers is denoted by e_j, for $j = h, l$ and the total measure of workers is normalize to one. We assume a complete set of insurance markets such that the worker's saving choices do not depend on its state on the labor market. Thus there is a representative household that solves the following problem:

$$Max\ E\left\{\sum_{t=0}^{\infty}\beta^t U(c_t, h_t)\right\} \tag{3}$$

where c_t denotes consumption and h_t denotes time spent at the work place. The specification of the utility adopted in our model is the following:

$$U(c_t, h_t) = \log(c_t) + n_{h,t}\Gamma_t^{n_h} + n_{l,t}\Gamma_t^{n_l} + u_{h,t}\Gamma_t^{u_h} + u_{l,t}\Gamma_t^{n_l}$$

where $\Gamma_t^{n_i} = \phi_{1i}\frac{(1-h_{i,t})^{1-\eta^u}}{1-\eta^u}$ and $\Gamma_t^{u_i} = \phi_{2i}\frac{(1-e_t)^{1-\eta^u}}{1-\eta^u}$. This function, is the one used in Andolfatto (1996). Although is not the standard specification in RBC models, it would allow us to analyze straightforward the implications of introducing the assumption of loss of skills during unemployment with respect to the "reference model" in the literature.

The household has to decide how to split current income between consumption and investment. Its income is made up of capital income, unemployment benefits and the wage bill net of the lump sum, Ψ_t, they have to pay to the government to finance the unemployment insurances, $b_{j,t}$. Therefore, the household's budget constraint in period t is

$$c_t + i_t + \Psi_t \leq w_{h,t} n_{h,t} h_{h,t} + w_{l,t} n_{l,t} h_{l,t} + u_{h,t} b_{h,t} + u_{l,t} b_{l,t} + r_t k_t + p_t$$

and investment is defined as follows:

$$i_t = \ell k_{t+1} - (1-\delta) k_t$$

Unemployment is a predetermined variable whose laws of motion are given by:

$$u_{h,t+1} = u_{h,t} - m_{h,t} + \sigma_h n_{h,t} - \gamma u_{h,t}$$

$$u_{l,t+1} = u_{l,t} - m_{l,t} + \sigma_l n_{l,t} + \gamma u_{h,t}$$

where $u_{j,t}$ denotes the measure of type j searchers, σ_j is the exogenous rate of job destruction and ϕ_j is the perceived probability that an unemployed worker be matched in period t. $\gamma u_{h,t}$ represents the fraction of workers that suffer from a loss of skill while unemployed. As we said, this process follows a Poisson distribution with parameter γ. Like we have normalized our population to one, this means that every period a fraction γ of the high skilled workers suffer a "decapitalization" while becoming long term unemployed.

Optimal contract - Following the standard literature on frictional unemployment, we assume that wages are the solution to a Nash-bargaining problem. Hence, if we denote as p_j the worker's bargaining power, the optimal contract is given by

$$w_{j,t} h_{j,t} = \arg\max \left\{ W_{nj,t}^{p} J_{nj,t}^{(1-p)} \right\}, \quad \text{for } j = h, l$$

where $W_{n_j,t}$ and $J_{n_j,t}$ represent the income value of employment of type j to the household and the firm respectively. p_j will be treated as a constant parameter strictly between 0 and 1.

The income value of high-skill employment to the household in units of the consumption good is given by:

$$W_{n_h}(\Omega_t^H) = \lambda_t w_{h,t} h_{h,t} + (\Gamma^n - \Gamma^u) + \beta E_t\left[W_{n_h}(\Omega_{t+1}^H)\right](1-\phi_h - \sigma_h)$$
$$+\gamma\beta\left(E\left[W_{nl}(\Omega_{t+1}^H)\right] - E_t\left[W_{n_h}(\Omega_{t+1}^H)\right]\right)$$

and is made up of three components: (i) the household gain in terms of wages because an additional high-skill agent starts working (ii) the loses in terms of leisure that this new job generates and (iii) the expected present value of this job in the future. This expected present value is formed by the continuation value of the job, that is the net probability of keeping a high-skill employment, minus the probability γ that the high-skill worker suffer from a depreciation of his human capital and become a low-skilled worker

The minimum wage the worker is going to accept comes from $W_{n_h}(\Omega_t^H) = 0$. Notice that the high skilled worker takes into account the possibility of becoming long term unemployed and with probability γ loosing some of his skills, therefore he is willing to accept a lower wage than in the standard case in which $\gamma = 0$.

Similarly, the firm's marginal benefit from employment is made up of the job's yields, i.e., the contribution to output of this marginal worker minus the returns to his work, plus the expected present value of this job in the future.

$$J_{nh}(\Omega_t^F) = F_{nh,t} - w_{h,t} h_{h,t} + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} J_{ns}(\Omega_{t+1}^F)\right](1-\sigma_h)$$

For low-skill workers, the household's marginal value of low skill employment is given by the equation below, which includes the net gain in utility for an additional low skill worker plus the expected present value of this job in the future.

$$W_{nl}(\Omega_t^H) = (\Gamma^n - \Gamma^u) + \lambda_t w_{l,t} h_{l,t} + \beta E_t\left[W_{nl}(\Omega_{t+1}^H)\right](1-\phi_{l,t} - \sigma_l)$$

The firms' marginal value of low skill employment equals the job's yields plus the expected present value of this position in the future.

$$J_{nl}(\Omega_t^F) = F_{nl,t} - w_{l,t}h_{l,t} + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}J_{nl}(\Omega_{t+1}^F)\right](1-\sigma_l)$$
$$+\eta\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[J_{ns}(\Omega_{t+1}^F) - J_{nl}(\Omega_{t+1}^F)\right]\right\}$$

notice that this expected present value includes the probability η of transforming the low quality match into a high quality one. $J_{nh}(\Omega_t^F) = 0$, gives the maximum compensation that the employer is willing to pay[3].

From the first order condition of this maximization problem, we can know that the worker will get a share p_j of the total income generated by the match.

$$(1-p_j)\frac{1}{\lambda_t}W_{n_j,t} = p_j J_{n_j,t}$$

Therefore, the optimal contract for each type of worker in this economy is given by the reservation wage and a fraction p of the net surplus they create by accepting the job offer. Net surplus means the product value net of what workers give up in terms of leisure and reservation wage. These optimal contracts can expressed as follows:

$$w_{h,t}h_{h,t} = p_h F_{nh,t} + p_h a_h \frac{v_{h,t}}{u_{h,t}} + p_h\gamma\left(\frac{a_h}{q_{h,t}} - \frac{a_l}{q_{l,t}}\right) + (1-p_h)b_{h,t}$$
$$-(1-p_h)c_t\left(\phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} - \phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)}\right)$$

The reservation wage of a high-skilled worker is given by the unemployment insurance plus the leisure in terms of utility enjoyed by the potential worker. The net surplus is given by the contribution of the worker to the output, which is his marginal productivity plus the savings in terms of posting vacancies' cost and the opportunity cost for the firm of not hiring the high skilled worker given that with

[3]Note that if the firm offers this maximum compensation to the worker, it would generate negative profits in the steady state, because it does not take into account the fact that posting vacancies is a costly activity.

probability γ he can suffer a depreciation of skills net of what the worker gives up which is his reservation wage.

Similarly, the optimal contract for the low-skill worker is given by the following equation:

$$\begin{aligned} w_{l,t}h_{l,t} &= p_l F_{nl,t} + p_l a_l \frac{v_{l,t}}{u_{l,t}} + p_l \eta \left(\frac{a_h}{q_{h,t}} - \frac{a_l}{q_{l,t}} \right) + (1-p_l) b_{l,t} \\ & -(1-p_l) c_t \left(\phi_{1,l} \frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} - \phi_{2,h} \frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} \right) \end{aligned}$$

The only difference with respect to the optimal contract for the other type of workers is that now, the firms takes into account that when it hires a low-skill worker he can become high-skill type with probability η.

To disentangle wages and hours worked we need two additional equations. We can compute the optimal number of hours for each type of worker differentiating the total surplus of each type of match $S_{j,t} = \frac{1}{\lambda_t} W_{n_j,t} + J_{n_j,t}$ with respect to the hours $\frac{\partial S_{j,t}}{\partial h_{j,t}}$, so that the optimal number of hours worked for each type of worker can be represented as:

$$\phi_1 \frac{1}{\lambda_t} (1-h_{j,t})^{(-\eta_j)} = (1-\theta) \frac{y_t}{l_t}$$

Definition of the recursive equilibrium - We can define the equilibrium of this economy as a set of infinite sequences for the rental price of capital $\{r_t\}$, wage rates $\{w_{h,t}, w_{l,t}\}$, employment and unemployment levels $\{n_{h,t}, n_{l,t}, u_{h,t}, u_{l,t}\}$, capital $\{k_t\}$, consumption $\{c_t\}$, vacancies $\{v_{h,t}, v_{l,t}\}$, hazard rates for workers $\{\phi_{h,t}, \phi_{l,t}\}$ and vacancies $\{q_{s,t}, q_{l,t}\}$, such that,

(i) Taking the rental prices and matching rates as given, $\{k_t\}$, $\{n_{h,t}, n_{l,t}\}$ and $\{v_{h,t}, v_{l,t}\}$ maximize the firms' profits

(ii) Wages are the solution to the Nash bargaining problem

(iii) Taking wages, the rental price of capital and hazard rates, $\{c_t\}$ and $\{k_t\}$ solves the household optimization problem

(iv) Hazard rates are given by the matching function

4 Estimation of the model

The model presented above has a large number of parameters. This rises the problem of assigning values to all of them. Standard calibration does not seem the best technique when the models are richly parameterized given that neither the focus on a limited set of moments of the model nor the transfer of microeconomics estimates from one model to another will provide the discipline to quantify the behavior of the model; so we have to rely in alternative methods that allow us to properly estimate the parameters of the model. We will estimate the parameters of our model via Maximum Likelihood.

Maximum Likelihood provides a systematic procedure to give values to all the parameters of interest. This means that we have to evaluate the likelihood function of our DSGE model. Except in a few cases, there is no analytical or numerical procedure to directly do it. But we can transform the theoretical model into a state-space econometric model and under the assumptions that the shocks to the economy are normally distributed and the linear approximations of the policy functions of the model, we can look for a numerical approximation of the likelihood function with the help of the Kalman filter. In what follows we explain this in more detail.

State-space representation of the model - Appendix A describes the competitive solution to the model above, so that when $\varepsilon_t^a = 0$, the economy converges to a steady state in which each of these detrended variables remains constant. This steady state, depends on some parameters describing tastes, technologies and matching. The appendix B contains the log-linearizations around the steady state from which we will implement the method proposed by Uhlig (1997) that when applied to a linear system yields the approximate solution or policy rules of the form

$$x_t = Ax_{t-1} + B\varepsilon_t$$
$$y_t = Cx_t$$

Where x_t and y_t represent vectors of logarithmic deviations of the states and the control variables from their steady-state levels. The elements of the matrices A,B and C depend on some of the model structural parameters.

The solution above considers that there is only one shock, the technology shock, driving the business cycle. This makes the model stochastically singular, i.e., the model predicts that certain combinations of the endogenous variables will hold with equality, and if in the data these exact relationships do not hold, maximum likelihood estimation will not be a valid method for the estimation.

Therefore, we should do any transformation in the model that allow us to overcome this problem. As Ireland (2004) explains, there are two common approaches to face the stochastic singularity problem. The first one consists in increasing the number of shocks in the model until we have the same number of shocks as number of time series used in the estimation; whereas the second approach, which is the one we use here, consists in augmenting last equation of the system above with an error term or measurement error, me_t. These errors represent the movements in the data that the theory does not explain (those movements that are not generated by the TFP shock, in our case) and are uncorrelated across variables. Then we have a system of the form:

$$s_t = Fs_{t-1} + V\varepsilon_t$$

$$f_t = Gs_t + me_t$$

where f_t denotes a vector of variables observed at date t, and s_t is the state vector, F and G are again matrices of parameters. The first equation of the system is known as the state equation and the second is known as the observation equation. Vectors ε_t and me_t are white noise vectors with $E[\varepsilon_t \varepsilon_t'] = Q$ and $E[me_t me_t'] = R$. Also $E[\varepsilon_t me_t'] = 0$

The advantage of this approach with respect to the first one, in which we have to include additional shocks relies on the fact that no restrictions are imposed. Otherwise, as Ireland (2004) says "*it would have required to lean even harder in* *economic theory by making further and specific assumptions about the behavior of the economy*".

Thus, once we have included this measurement errors, our theoretical DSGE model takes the form of a state-space econometric model whose parameters can now be estimated via maximum likelihood[4].

Kalman filter and approximation of the likelihood function - The empirical model written as a state-space econometric model, allows for the evaluation of the likelihood function using the Kalman filter algorithm explained in detail in Hamilton (1994, Chapter 13).

The ultimate objective is to estimate the values of the unknown parameters in the system on the basis of these observations $f_1, f_2, ..., f_T$. The Kalman filter works as a recursive estimator that takes initial values for the state-vector $\widehat{s}_{t|t-1}$ and its associate mean squared error $P_{t|t-1}$,to calculate linear least square forecast of the state-vector for subsequent periods t=2,3,... T. This forecasts are of the form. $\widehat{s}_{t|t-1}$=$\widehat{E}[s_{t+1}| f_t]$., where $\widehat{E}[s_{t+1}| f_t]$ is the linear projection of s_{t+1} on f_t and a constant. The Kalman filter has two main phases: prediction and update.

In the prediction phase, using the law of iterated projections, it plugs the forecast $\widehat{s}_{t|t-1}$ into the observable equation to yield a forecasting of f_t

$$\widehat{f}_{t|t-1} = G\widehat{s}_{t|t-1}$$

the error of this forecast is defined as $w_t = f_t - \widehat{f_{t|t-1}} = Gs_t + me_t - G\widehat{s}_{t|t-1} - me_t$ with MSE $E[(f_t - \widehat{f_{t|t-1}})(f_t - \widehat{f_{t|t-1}})'] = FP_{t|t-1}F' + R$.

In the second phase, the inference about the current value of s_t is updated on the basis of the observation of f_t to produce $\widehat{s}_{t|t}$.. Introducing it into the state equation produces a forecast of $\widehat{s}_{t+1|t}$

[4]Ireland (2004) calls it "hybrid model" because it combines the power of the DSGE models with the flexibility of a VAR.

$$\widehat{s}_{t+1|t} = F\widehat{s}_{t|t} + 0$$

evaluating $\widehat{s}_{t|t}$ by using the formula of updating a linear projection, and substituting it above, we get the best forecast of s_{t+1} based on a constant and a linear function of the observable vector f_t

$$\widehat{s}_{t+1|t} = F\widehat{s}_{t|t-1} + K_t(f_t - \widehat{f}_{t|t-1})$$

where K_t is the optimal Kalman gain matrix, which depends on the matrices F, G, R and the stationary variance Σ_t. Those matrices are not function of the data but entirely determined by the population parameters of the process. $\widehat{s}_{t+1|t}$ denotes the best forecast of st+1 based on a constant and a linear function of the observables f_t if and only if K_t is the optimal gain matrix.

The application of the Kalman filter let us calculate the log-likelihood function of the hybrid model as

$$ln(L) = -\frac{3T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|G\Sigma_t G'| - \frac{1}{2}\sum_{t=1}^{T} w_t' \left(G\Sigma_t G'\right)^{-1} w_t$$

Using a numerical search algorithm one can find the set of parameters contained in the matrices F, G, Q and R that maximize the likelihood function. Usually, maximum likelihood estimations of this type are criticized because it is very difficult to be sure whether we are in the global maximum or on the contrary we are just in a local one, given that the likelihood function displays a quite sinuous pattern. To avoid this criticism with our estimations we borrow from physics another algorithm called "*simulated annealing*" . This is a generic probabilistic meta-algorithm for the global optimization problem, i.e. it looks for a good approximation to the global optimum of a function in a large search space. Each step of the simulated annealing algorithm replaces the current solution by a random "nearby" solution. The allowance for these movements saves the method from becoming stuck at a local minimum.

In principle, this numerical algorithm is allowed to select values of the parameters that lie anywhere between the positive and the negative infinity. But to ensure that our parameters satisfy the theoretical restrictions listed in section 2, additional constraints have been imposed[5].

The data - Data is taken from the Federal Reserve Bank of St. Louis' FRED database. Data for gross domestic income and wages is taken from the Bureau of Economic Analysis. In the appendix C, we present detailed information about each series. Monthly data has been transformed into quarterly data using averages. The period selected goes from 1964-1 to 2005-4.

When the model takes the form of a state-space representation, it can be estimated via maximum likelihood once analogs to the model's variables are found in the data. Therefore, C_t is defined as real personal consumption on non durables and services plus government expenditure. Investment, I_t is defined as the sum of consumption on durable goods plus investment. Vacancies, V_t are proxied by a widely used index which reflects the number of "*help-wanted*" advertisement registered in US newspapers. N_t comes directly from the number of civilian employment, and thus unemployment can be computed as $1 - N_t$.

All the variables have been divided by the civilian population aged 16 or over, so as to have them in per capita terms. On top of that we have taken logarithms of all the variables and calculated the growth rates when necessary. Series have not been filtered in any other way.

Finally, to make them comparable with the vectors of logarithmic deviations of the variables from their steady-state levels, we have to use the definitions:

[5]In particular, some of ours parameters are constrained to be positive, so we constraint the algorithm to work with absolute values. Many of ours parameters are probabilities that should lie between zero and one, so we again constraint the algorithm to work with the logistic transformation of the parameter.

$$\widehat{c_t} = \log(c_t) - \log(\bar{c})$$

$$\widehat{i_t} = \log(i_t) - \log(\bar{i})$$

$$\Pr od_t = \log(\Pr od_t) - \log(\overline{\Pr od}) - t\log(\ell)$$

$$\widehat{t_t} = \log(t_t) - \log(\bar{t})$$

$$\widehat{\theta_t} = \log(\theta_t) - \log(\bar{\theta})$$

for all t=1....T. Remember that to solve the problem we have normalized output to 1. Therefore, consumption and investment here are defined as $\widehat{c_t} = \widehat{\left(\frac{c_{d,t}}{y_t}\right)}$. and $\widehat{i_t} = \widehat{\left(\frac{i_{d,t}}{y_t}\right)}$. Productivity is growing at rate ℓ in steady state. Once those transformations are made, the vector of observable is given by:

$$f_t = \begin{bmatrix} \widehat{c_t} & \widehat{i_t} & \widehat{t_t} & \widehat{Pd_t} & \widehat{\theta_t} \end{bmatrix}$$

Parameter estimates - Usually algorithms for computing maximum likelihood estimates have the drawback that they do not produce standard errors. This means that we should look numerical approximations of the derivatives of the likelihood function so as to compute the information matrix and then the standard errors.

Fortunately, we know that if certain regularity conditions hold[6], the maximum likelihood estimates are consistent and asymptotically normal. Under these circumstances, the information matrix for a sample of size T can be calculated from the second derivatives of the maximized log-likelihood function as

[6]These conditions include that the model must be identified, the eigenvalues of A are inside the unit circle, the true values of the estimations do not fall on a boundary of the allowable parameter space and that variables x_t behave asymptotically like a full-rank linearly indeterministic covariance-stationary process

$$I_T = -\frac{1}{T}\left\{\sum_{t=1}^{T} \frac{\partial^2 \log L(y_t,\theta)}{\partial\theta\ \partial\theta'}\right\} \tag{4}$$

Standard errors are then the square roots of the diagonal elements of $\frac{1}{T}(I_T)^{-1}$. This matrix has elements of very different magnitudes and therefore, the reported standard errors should be interpreted with caution.

Results of the estimation - In the next table we report the maximum likelihood estimations for the parameters and their standard errors. The values of the parameters estimated constitute all of them sensible results.

The first thing worth noting is the slow upgrade of skills. This result is in line with the assumption made by the turbulence hypothesis presented in Ljungqvist and Sargent's series of papers and has been widely challenged by Den Haan, Ramey and Watson (2001 and 2005) in different papers because what they think is too low upgrading of skills. What we see is that the data does not give support to this criticism and supports the value proposed by LS.

Data does not fulfill the efficiency condition or Hosios condition which says that the bargaining power or the workers pj should equal the parameter (1-αj) of the matching function so as to have efficient results. As we can see our estimations do not present evidence in favor of this condition neither support the proposal of Hagedorn and Manovskii (2005)

As we have said before in most cases we have had to impose additional constraints when estimating the parameters of the model. Usually those constraints are related to the fact that many of the parameters are probabilities whose value should lie between zero and one. One additional constraint has been related to the parameter η^u of the utility function. If we let this value free it reaches values close to 20. This value seems extremely large and we have constrained it to lie between 0 and 5.

Table 2: Estimated parameters and standard deviations

Parameter	Value	Explanation	Std. error
α_h	0.80	elasticity vacancies type h	0.0010
α_l	0.80	elasticity vacancies type l	0.0021
η	0.01	upgrade of skills	0.0040
ρ	0.95	technology persistence	0.0119
ε	0.01	variance	0.0001
p_h	0.10	bargaining power workers	0.0009
p_l	0.355	bargaining power workers	0.0007
g	1.005	deterministic growth rate	0.0001
θ	0.583	elasticity of capital	0.0027
δ	0.03	depreciation of capital	0.0059
τ	0.90	efficiency units of low product workers	0.0124
σ_h	0.0781	exog. destruc rate for h	0.0027
σ_l	0.0897	exog. destruc rate for l	0.0020
η^u	3.3341	parameter utility function	0.0009

The deterministic growth rate of the economy takes a value close to one. This means that we can perfectly work with a stationary model. The less satisfactory results are for the values driving the investment and capital accumulation of the economy. We obtain a value of θ close to 0.6 whereas the standard value in the literature is close around 0.4. Also the depreciation of capital, δ, takes a higher value than what is standard. This has led us to estimate a second version of the model, in which we have constrained the values of those parameters to take the standard values. The performance of variables such as consumption and investment improves with this new specification as we will see afterwards.

It is also quite common to obtain low estimates for the discount factor β, showing the preference of the household for consumption today. What we have done is to estimate the parameters of the model keeping β fixed and equal to 0.99.

The remainder parameters of the model can be obtained through the estimations above Table 3 describes those parameters and shows the values they adopt. Again all of them appear quite reasonable

Table 3: Parameters that can be obtained through the estimations

Parameter	Value	Explanation
γ	0.0265	downgrade (prob of becoming long term)
χ^s	0.8499	effic paramet. matching funct
χ^l	0.8526	effic paramet matching funct
ξ	0.5585	parameter production funct.
a_h	0.1856	cost of posting a high produc.vacancy
a_l	0.1856	cost of posting a low produc.vacancy
$\phi_{1,h}$	1.4260	parameter in utility function
$\phi_{1,l}$	0.7130	parameter in utility function
$\phi_{2,h}$	2.9669	parameter in utility function
$\phi_{2,h}$	0.8157	parameter in utility function

5 Results

This section is divided in two parts. In the first one we present the quantitative properties of our model whereas in the second we analyze the contribution of our model to the literature reviewed in section 2 that tries to explanin the rise in European unemployment and its persitencew over time.

5.1 Evaluation of the model

Table 4 presents the volatilities of the main variables of the model and compares them with the values for the US economy and the Andolfatto (1996) model. We have two columns of results: DSGE(1) and DSGE (2). The first one corresponds to the values of the parameters presented in Table 3 above whereas DSGE(2) represents the more constrained model explained above. In particular, we have proceed again with the estimation of the main parameters of the model but keeping the values of β, θ and δ fixed.

What we can see from these values is that the volatility of total hours worked, employment, hours per worker and tightness increase substantially with respect to the Andolfatto model. Our model increase also the volatility of both unemployment and vacancies with respect to the search economy or the Andolfatto results. We can see this as a success given the large literature dealing with it nowadays.

The model has some difficulty in explaining the larger volatility of the variable hours per worker, which results substantially larger in the model than in the data. Accordingly, the volatilities of the wage bill and the labor share of output also result more volatile than in the data. This result depends to a great extent on the parameter η^u of the utility function that we have not allow to take larger values than 5. Probably, a more standard specification of the utility function could yield more satisfactory results concerning the performance of this variable or even allowing for this parameter to take larger values can partially solve the problem.

The DSGE(2) yields overall better results in terms of volatilities than DSGE(1). We do not see any inconvenient in dealing with these model instead with the less restricted one. The performance of the investment, consumption and tightness improves significantly when allowing for those constraints while the remaining values are pretty similar to those obtained under the DSGE(1).

Table 4: Volatilities

Variable	US Economy	RBC Search	DSGE (1)	DSGE (2)
Consumption	0.56	0.32	0.27	0.32
Investment	3.14	2.98	2.08	3.12
Total hours	0.93	0.59	0.80	0.75
Employment	0.67	0.51	0.72	0.70
Hours\Worker	0.34	0.22	1.06	1.07
Tightness		3.30	4.61	6.78
Wage bill	0.97	0.94	1.86	1.89
Labor share	0.68	0.10	1.17	1.00
Productivity	0.64	0.46	0.27	0.32
Real Wage	0.44	0.39	0.26	0.27

One of the major interests of the theoretical model presented in this paper is that if satisfactory, it can be used to analyze European labor markets. One common characteristic of those markets is the existence of long-term unemployment or, in other words, high persistence of unemployment. The standard RBC model with frictional labor markets, although satisfactory in replicating the persistence of US unemployment, have major problems when replicating the persistence of European labor markets (see Esteban-Pretel, 2004). When we introduce the assumption of loss of skills during unemployment in combination with unemployment benefits, the persistence of unemployment increases substantially. For example, under the reference Andolfatto model, 86 per cent of the unemployed workers that lose their jobs remain unemployed one quarter apart whereas only 18 per cent of them remain unemployed within a year. Under our specification almost half of them remain unemployed within a year, which constitutes almost 70 per cent of the unemployed workers who have suffered from a depreciation of skills.

Table 5: Persistence of unemployment

Variable (x)	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$	$x(t+4)$
Search Economy					
Total unemployment	1	0.86	0.63	0.39	0.18
Turbulent workers	–	–	–	–	–
DSGE (1)					
Total unemployment	1	0.93	0.80	0.63	0.45
Turbulent workers	1	0.98	0.91	0.80	0.67
DSGE (2)					
Total unemployment	1	0.94	0.80	0.63	0.45
Turbulent workers	1	0.98	0.91	0.80	0.67

5.2 Implications for the labor market performance

As we have seen at the begining of this paper, the turbulence hypothesis relies on the combination of microeconomic shocks to the human capital of unemployed workers and labor market institutions to explain the steadily increase in unemployment in Europe from the late 1970s onwards. In particular, Ljungqvist and Sargent (1998) central assumption is that "*the last couple of decades saw an increased probability of human capital loss at the time of an involuntary job displacement*". This increase in the probability of losing skills in combination with generous unemployment benefits, produce high long-term unemployment.

Given that the values of the estimated parameters are in line with the values proposed by LS, we can split the initial sample into two sub-samples and analyze whether the data gives support to the hypothesis of an increase in turbulence or not. Therefore, we split the sample into two disjoint sub-samples. The first one covers the period 1964Q1-1980Q1, and the second runs the rest of the sample, i.e. 1980Q2-2005Q1. The breakpoint corresponds to the date around which we consider the European unemployment started rising[7].

Then we re-estimate the parameters and check for their stability. Across the board, the test reject the null hypothesis of parameter stability. This result is in line with previous findings of Ireland (2004) and Altug (1989) and Stock and Watson (1996) who also found evidence of instability in parameters in RBC and VAR models respectively.

But what is more interesting for our analysis, is that the parameter γ that measures the probability of human capital loss remains constant along the two subsamples and for both specifications of the model - without and with constraints on some of the parameters-.

Period	Parameter
t_1= $1964Q1 - 1980Q1$	γ_1=0.0265
t_2= $1980Q2 - 2005Q4$	γ_2=0.0265
Conclusion	γ_1= γ_2

We see this result very interesting as far as it contributes to the literature on unemployment in the following way: According to our model and the estimation of its parameters, we do not observe evidence in favor of an increase in the probability of human capital loss. This does not mean that we find the LS explanation wrong because, in spite of the fact that we do not observe an increase in the value of this parameter, the turbulence hypothesis is necessary to improve the performance of the RBC model and, more important for the understanding of unemployment, it is able to increase the persistent unemployment and consequently generate long-term unemployment. This is a characteristic that previous RBC models embedding matching models in the labor market were unable to produce.

[7] We have tried different breakpoints running from 1975Q4 to 1982Q4, giving all of them similar results.

Stated in other words, we explain the increase in unemployment in the following terms: We belive that there is a negative aggregate shock that increases the pool of unemployed workers. This makes the fraction of turbulent workers increase- not because the probability of losing skills increases but because the initial pool has become bigger due to an aggregate shock-. At this point we have an increase in the number of turbulent unemployed workers who have loss their initial skills but receive unemployment benefits that correspond to the high skill level. The rest of the story is the LS one.

In a way, our results conciliate the macro and micro explanations proposed in the literature to explain the increase in unemployment and its persistence over time.

6 Conclusion

In this paper we have proposed a DSGE model in which the labor market is explicitly modellized as a frictional market where firms and workers engage in productive job matches. We have also introduced the assumption of loss of skills during unemployment to analyze the hypothesis of turbulence proposed in the literature to explain the steadily increase in the European unemployment since the late 1970s.

We have then brought the model to the data. We have estimated the parameters of the model via maximum likelihood and we have seen that the values obtained are in line with those proposed by LS. In particular, we find very low probability of upgrading skills for the already matched workers and similar value for the turbulence parameter. We have also analyzed the quantitative properties of our model and we have studied the stability of the probability of suffering from a depreciation of human capital.

We find that the data does not give evidence of an increase of the probability of losing skills once the match have been dissolved, contrary to the central assumption of LS's explanation. Nevertheless, allowing for this depreciation of the human capital, generates long-term unemployment or a very strong persistence of the unemployment for all workers in the economy and mainly for those who have suffered the loss of skills.

Wrapping up, we conclude that according to our model we cannot accept the assumption that an increase in the turbulence occurred at the end of the 70s. Instead we find evidence on the existence of a negative aggregate shock that increased the pool of unemployed workers. We do not see this as a rejection of subsequent explanation given by LS, i.e. that we need the combination of the loss of skills assumption and generous unemployment benefits to produce long-term unemployment and be able to explain the steadily increase in unemployment over time.

In a way, our model conciliates the two streams of the literature that have tried to explain the European unemployment dilemma based on the one hand on the combination of macroeconomic shocks and labor market institutions and the combination of micro shocks and those labor market institutions, on the other hand.

References

[1] Altug, S. (1989): "Time to built and aggregate fluctuations: some new evidence", International Economic Review, Vol.30, N.4, 889-920

[2] Andolfatto, D (1996): "Business Cycles and Labor Market Search". *American Economic Review* ,86 (1), 112-132

[3] Blanchard O. and Diamond (1994): "Ranking, Unemployment Duration, and Wages". *Review of Economic Studies* 61 (3), 417-434

[4] Blanchard O. and J. Wolfers (1999): "The role of shocks and institutions in the rise of European unemployment". *NBER Working Paper* No. W7282

[5] Blanchard O. and J. Wolfers (2000): "The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence". *The Economic Journal*, 110 (462), C1-C33.

[6] Bover O., M. Arellano and S. Bentolila (2002): "Unemployment Duration, Benefit Duration and the Business Cycle". *The Economic Journal*, 112, 223-265.

[7] Cheron, A. and F. Langot ():"Labor market search and Real Business Cycles: reconciling Nash bargaining with the real wage dynamics", *Review of Economic Dynamics*, 7, 476-493

[8] Hamilton, (1994): Time series econometrics, Ch.13

[9] Hansen, L., E. McGrattan and T. Sargent (1994): "Mechanics of Forming and Estimating Dynamic Linear Economies". *Federal Reserve Bank of Minneapolis. Research Department. Staff Report* 182.

[10] Hosios, A.J. (1990): "On the efficiency of matching and related models of search and unemployment". *Review of Economic Studies* 57 (3), 279-298

[11] Ireland, P. (2004): "A method for taking models to the data". Journal of Economic Dynamics and Control, 28,1205-1226

[12] Jackman, R and R. Layard (1991): "Does long-term Unemployment Reduce a Person's Chance of a Job? A Time Series Test", *Economica*, 58 (229), 93-106

[13] Layard, R., S. Nickell and R. Jackman (1991): *Unemployment, Macroeconomic Performance and the Labour Market*, Oxford, Oxford University Press.

[14] Ljungqvist, L. and T. Sargent (1998):"The European Unemployment Dilemma", *Journal of Political Economy*, 106, 514-550.

[15] Ljungqvist, L. and T. Sargent (2002): "The European Employment Experience", CEPR Discussion Paper No. 3543

[16] Ljungqvist, L. and T. Sargent (2004): " European Unemployment and Turbulence revisited in a matching model", CEPR Discussion Paper No. 4183

[17] Ljungqvist, L. and T. Sargent (2005): " Jobs and Unemployment in Macroeconomic Theory: A turbulence laboratory", CEPR Discussion Paper No. 5340

[18] Den Haan, W. G. Ramey and J.Watson (2000):"Job destruction and propagation of shocks", *American Economic Review*, 90(3), 482-498

[19] Den Haan, W, C. Haefke and G. Ramey (2001): "Shocks and Institutions in a Job Matching Model", NBER Working Papers, No. 8463.

[20] Den Haan, W, C. Haefke and G. Ramey (2005): "Turbulence and unemployment in a job matching model", NBER Working Papers, No. 8463.

[21] Merz, M (1995):"Search in the labor market and the real business cycle", *Journal of Monetary Economics*, 36, 269-300.

[22] Nickel, S. (1979),"Estimating the probability of leaving unemployment" *Econometrica* 4(5), 1249-1266

[23] Petrongolo, B. and C. Pissarides (2001): "Looking into the Black Box: A survey of the matching function". *Journal of Economic Literature* 39, 390-431.

[24] Phelps, E. (1994): *Structural Slumps. The modern equilibrium theory of unemployment, interest and assets.* Cambridge MA. Harvard University Press.

[25] Pissarides, C. (1992): "Loss of skill during unemployment and the persistence of employment shocks", *Quarterly Journal of Economics*, 107(4),1371-1391

[26] Pissarides, C. (2000): *Equilibrium Unemployment Theory.* The MIT Press, Cambridge. Second Edition

[27] Prescott, E. (2004): Why do Americans work so much more than Europeans?". *Federal Reserve Bank of Minneapolis Quarterly Review*, 28 (1), 2-13.

[28] Rebelo, S (2005): "Real Business Cycle Models: Past, Present and Future", mimeo

[29] Ravn, M (2005): "Labor Market Matching, Labor Market Participation and Aggregate Business Cycles: Theory and Structural Evidence for the US", European University Institute, mimeo

[30] Ravn, M (2006): "The Consumption-Tightness Puzzle", European University Institute, mimeo

[31] Ruge-Murcia, F. (2003): "Methods to Estimate Dynamic Stochastic General Equilibrium Models", Université de Montréal, cahier 2003-23.

[32] Shimer, R (2005): "The Cyclical Behaviour of Equilibrium Unemployment and Vacancies", *American Economic Review* 95(1), 25-49

[33] Uhlig, H. (1997): "A toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily"

A Appendix: solving for the non-stochastic equilibrium

A.1 Households

The household maximizes the following problem

$$W(\Omega_t^H) = \max_{c,k'} \left\{ \begin{array}{l} \log(c_t) + n_{h,t}\phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} + u_{h,t}\phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} \\ + n_{l,t}\phi_{1,l}\frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} + u_{l,t}\phi_{2,l}\frac{(1-e)^{(1-\eta_l)}}{(1-\eta_l)} + \beta E_t W(\Omega_{t+1}^H) \end{array} \right\}$$

subject to the budget constraint and the law of motion for capital

$$c_t + i_t + \Upsilon_t = w_{h,t}n_{h,t}h_{h,t} + w_{l,t}n_{l,t}h_{l,t} + u_{h,t}b_{h,t} + u_{l,t}b_{l,t} + r_t k_t + \Pi_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

where $u_{l,t} = u_{t,t} + u_{b,t}$ and $b_{l,t} = \frac{u_{t,t}b_{h,t} + u_{b,t}b_{b,t}}{u_{l,t}}$

The first order conditions of the problem and the envelope theorem are presented below:

w.r.t. consumption $\quad \frac{1}{c_t} = \lambda_t$

w.r.t. capital $_{t+1}$ $\quad \beta E_t \left[W_k(\Omega_{t+1}^H) \right] = \lambda_t$

budget constraint $\quad c_t + k_{t+1} - (1-\delta)k_t = w_{h,t}n_{h,t}h_{h,t} + w_{l,t}n_{l,t}h_{l,t} + u_{h,t}b_{h,t} + u_{l,t}b_{l,t} + r_t k_t + \Pi_t$

envelope theorem $\quad W_k(\Omega_t^H) = \lambda_t \left[(1-\delta) + r_t \right]$

and yield the optimal behavior of the household. Therefore, the **optimal decisions for the households** are fully summarized by the following equations:

$$1 = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left((1-\delta) + r_{t+1} \right) \right\}$$

$$c_t + i_t + \Upsilon_t = w_{h,t} n_{h,t} h_{h,t} + w_{l,t} n_{l,t} h_{l,t} + u_{h,t} b_{h,t} + u_{l,t} b_{l,t} + r_t k_t + \Pi_t$$

$$k_{t+1} = (1-\delta) k_t + i_t$$

A.2 Firms

The Bellman equation that represents the problem of the firm can be stated as follows:

$$J(\Omega_t^F) = \max_{k, n'_h, n'_l, v_h, v_l} F(\xi a_t k_t^{\theta} L_t^{(1-\theta)} - w_{h,t} n_{h,t} h_{h,t} - w_{l,t} n_{l,t} h_{l,t} - r_t k_t - a_h v_{h,t} - a_l v_{l,t} + \beta E \left[\frac{\lambda_{t+1}}{\lambda_t} J(\Omega_{t+1}^F) \right]$$

subject to the laws of motion for employment and the AR process defining the technology

$$n_{h,t+1} = n_{h,t}(1-\sigma_h) + q_{h,t} v_{h,t} + \eta n_{l,t}$$

$$n_{l,t+1} = n_{l,t}(1-\sigma_l) + q_{l,t} v_{l,t} - \eta n_{l,t}$$

$$\log a_{t+1} = \rho \log a_t + (1-\rho) \log \overline{a} + \varepsilon_{t+1}^a$$

The first order conditions of the problem are presented below:

w.r.t capital $(1-\theta)\frac{y_t}{k_t}= r_t$

w.r.t vacancies for high-skill workers $a_h= \Theta_{h,t}\ q_{h,t}$

w.r.t vacancies for low-skill workers $a_l= \Theta_{l,t}\ q_{l,t}$

w.r.t high-skill employment t+1 $\Theta_{h,t}= \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} J_{n_h}(\Omega^F_{t+1})\right]$

w.r.t low-skill employment t+1 $\Theta_{l,t}= \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} J_{n_l}(\Omega^F_{t+1})\right]$

law of motion for high-skill employment $n_{h,t+1}= n_{h,t}(1-\sigma_h) + q_{h,t}v_{h,t}+\eta n_{l,t}$

law of motion for high-skill employment $n_{l,t+1}= n_{l,t}(1-\sigma_l) + q_{l,t}v_{l,t}-\eta n_{l,t}$

And applying the envelope theorem we have:

for high-skill employment $J_{nh}(\Omega^F_t) = F_{n_h,t}-w_{h,t}h_{h,t}+\Theta_{h,t}(1-\sigma_h)$

for low-skill employment $J_{nl}(\Omega^F_t) = F_{n_l,t}-w_{l,t}h_{l,t}+\Theta_{l,t}(1-\sigma_l) + \eta(\Theta_{h,t}-\Theta_{l,t})$

Thus, **optimal decisions for the firms** are fully summarized by:

$$(1-\theta)\frac{y_t}{k_t}= r_t$$

$$\frac{a_h}{q_{h,t}}= \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left(F_{n_h,t+1}-w_{h,t+1}h_{h,t+1}+\frac{a_h}{q_{h,t+1}}(1-\sigma_h)\right)\right\}$$

$$\frac{a_l}{q_{l,t}}= \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left(F_{n_l,t+1}-w_{l,t+1}h_{l,t+1}+\frac{a_l}{q_{l,t+1}}(1-\sigma_l)+\eta\left(\frac{a_h}{q_{h,t+1}}-\frac{a_l}{q_{l,t+1}}\right)\right)\right\}$$

$$n_{h,t+1}= n_{h,t}(1-\sigma_h)+q_{h,t}v_{h,t}+\eta n_{l,t}$$

$$n_{l,t+1}= n_{l,t}(1-\sigma_l)+q_{l,t}v_{l,t}-\eta n_{l,t}$$

A.3 Matching

The standard specification of the matching function is the following:

$$m_{j,t}=\chi_j v_{j,t}^{\alpha_j}u_{j,t}^{1-\alpha_j}$$

and the companion probabilities of matching for workers and vacancies are respectively:

$$\phi_{j,t}=\frac{m_{j,t}}{u_{j,t}} \quad \text{and} \quad q_{j,t}=\frac{m_{j,t}}{v_{j,t}}$$

A.4 Optimal contract:

To compute the optimal contract we just have to apply the first order condition of the Nash bargaining problem, which can be stated as follows

$$(1-p_j)\frac{1}{\lambda_t}W_{n_j}= p_j J_{n_j}$$

The marginal value of high-skill and low-skill employment for the household comes from the following expressions

$$W_{n_h}(\Omega_t^H) = \left\{ \begin{array}{l} \phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} - \phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} + \lambda_t w_{h,t} n_{h,t} h_{h,t} - \lambda_t u_{h,t} b_{h,t} \\ +\beta E_t\left[W_{n_h}(\Omega_{t+1}^H)\right](1-\phi_h-\sigma_h) + \gamma\beta\left(E\left[W_{nl}(\Omega_{t+1}^H)\right] - E_t\left[W_{n_h}(\Omega_{t+1}^H)\right]\right) \end{array} \right\}$$

$$W_{n_l}(\Omega_t^H) = \left\{ \begin{array}{l} \phi_{1,l}\frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} - \phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} + \lambda_t w_{l,t} h_{l,t} - \lambda_t u_{l,t} b_{l,t} \\ +\beta E_t\left[W_{n_l}(\Omega_{t+1}^H)\right](1-\phi_{l,t}-\sigma_l) \end{array} \right\}$$

which together with the income values of employment for the firms, J_{n_h} and J_{n_l}, yield the following contracts:

$$w_{h,t}h_{h,t}=p_hF_{nh,t}+p_ha_h\frac{v_{h,t}}{u_{h,t}}+p_h\gamma\left(\frac{a_h}{q_{h,t}}-\frac{a_l}{q_{l,t}}\right)+(1-p_h)b_{h,t}-(1-p_h)c_t\left(\phi_{1,h}\frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)}-\phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)}\right)$$

$$w_{l,t}h_{l,t}=p_lF_{nl,t}+p_la_l\frac{v_{l,t}}{u_{l,t}}+p_l\eta\left(\frac{a_h}{q_{h,t}}-\frac{a_l}{q_{l,t}}\right)+(1-p_l)b_{l,t}-(1-p_l)c_t\left(\phi_{1,l}\frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)}-\phi_{2,h}\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)}\right)$$

the optimal values for the hours worked of each type of worker are obtained via the mutual surplus of the match

$$S_{j,t}=\frac{1}{\lambda_t}W_{n_j,t}+J_{n_j,t}$$

and yield the following results

$$\frac{\partial S_{h,t}}{\partial h_{h,t}}= -\phi_1\frac{1}{\lambda_t}(1-h_{h,t})^{(-\eta_h)}+(1-\theta)\frac{Y_t}{L_t}\left(1-\theta\frac{n_{h,t}h_{h,t}}{L_t}\right)=0$$

$$\frac{\partial S_{l,t}}{\partial h_{l,t}}= -\phi_1\frac{1}{\lambda_t}(1-h_{l,t})^{(-\eta_l)}+(1-\theta)\frac{Y_t}{L_t}\tau\left(1-\theta\tau\frac{n_{l,t}h_{l,t}}{L_t}\right)=0$$

A.5 Government

$$u_{h,t}b_{h,t} + u_{l,t}b_{l,t} = \Upsilon_t$$

A.6 Non-stochastic general equilibrium

The general equilibrium is defined as a set of functions$\{c, i, v_h, v_l, u_h, u_l, n_h, n_l, t, k,$ $w_h, w_l, h_h, h_l, m_h, m_l, y_t, \tau_t, b_{h,t}, b_{l,t}\}$, solution of the following system

$$c_t + i_t + a_h v_{h,t} + a_l v_{l,t} = y_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

$$u_{h,t+1} = u_{h,t} + \sigma_h n_{h,t} - m_{h,t} - \gamma u_{h,t}$$

$$u_{l,t+1} = u_{l,t} + \sigma_l n_{l,t} - m_{l,t} + \gamma u_{h,t}$$

$$u_{h,t}b_{h,t} + u_{l,t}b_{l,t} = \Upsilon_t$$

$$b_{h,t} = rr_h w_{h,t-1}$$

$$b_{l,t} = rr_l w_{l,t-1}$$

$$n_{h,t+1} = n_{h,t}(1-\sigma_h) + m_{h,t} + \eta n_{l,t}$$

$$n_{l,t+1} = n_{l,t}(1-\sigma_l) + m_{l,t} - \eta n_{l,t}$$

$$y_t = \xi A k^{\theta}(t_t)^{(1-\theta)}$$

$$t_t = n_{h,t}h_{h,t} + \tau n_{l,t}h_{l,t}$$

$$m_{h,t} = \chi_h v_{h,t}^{\alpha h} u_{h,t}^{1-\alpha h}$$

$$m_{l,t} = \chi_l v_{l,t}^{\alpha l} u_{l,t}^{1-\alpha l}$$

$$w_{h,t}h_{h,t} = p_h F_{nh,t} + p_h a_h \frac{v_{h,t}}{u_{h,t}} + p_h \gamma \left(\frac{a_h}{q_{h,t}} - \frac{a_l}{q_{l,t}} \right) + (1-p_h)b_{h,t} - (1-p_h)c_t \left(\phi_{1,h} \frac{(1-h_{h,t})^{(1-\eta_h)}}{(1-\eta_h)} - \phi_{2,h} \frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)} \right)$$

$$w_{l,t}h_{l,t} = p_l F_{nl,t} + p_l a_l \frac{v_{l,t}}{u_{l,t}} + p_l \eta \left(\frac{a_h}{q_{h,t}} - \frac{a_l}{q_{l,t}} \right) + (1-p_l)b_{l,t}(1-p_l)c_t \left(\phi_{1,h} \frac{(1-h_{l,t})^{(1-\eta_l)}}{(1-\eta_l)} - \phi_{2,h} \frac{(1-e)^{(1-\eta_l)}}{(1-\eta_l)} \right)$$

$$(1-\theta)\frac{y_t}{L_t}\left(1 - \theta \frac{n_{h,t}h_{h,t}}{t_t}\right) = c_t \phi_1 (1-h_{h,t})^{(-\eta_h)}$$

$$(1-\theta)\frac{y_t}{L_t}\tau\left(1 - \theta\tau \frac{n_{l,t}h_{l,t}}{t_t}\right) = c_t \phi_1 (1-h_{l,t})^{(-\eta_l)}$$

and the expectational equations

$$1 = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left((1-\delta) + (1-\theta)\frac{y_{t+1}}{k_{t+1}} \right) \right\}$$

$$\frac{a_h v_{h,t}}{m_{h,t}} = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left(F_{nh,t+1} - w_{h,t+1}h_{h,t+1} + \frac{a_h v_{h,t+1}}{m_{h,t+1}}(1-\sigma_h) \right) \right\}$$

$$\frac{a_l v_{l,t}}{m_{l,t}} = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \left(F_{nl,t+1} - w_{l,t+1}h_{l,t+1} + \frac{a_l v_{l,t+1}}{m_{l,t+1}}(1-\sigma_l) + \eta \left(\frac{a_h v_{h,t+1}}{m_{h,t+1}} - \frac{a_l v_{l,t+1}}{m_{l,t+1}} \right) \right) \right\}$$

Given that we also impose that $1 = n_{h,t} + n_{l,t} + u_{h,t} + u_{l,t}$ one of the laws of motion above is a linear combination of the others plus the condition above and we have to take this into account

B Log-linearized equations

B.1 Deterministic equations

$$0 = c^* c(t) + i^* i(t) + a_h v_h^* v_h(t) + a_l v_l^* v_l(t) - y^* y(t)$$

$$0 = k^* k(t) - (1-\delta) k^* k(t-1) - i^* i(t)$$

$$0 = n_l^* n_l(t) - (1 - \sigma_l - \eta) n_l^* n_l(t-1) - m_l^* m_l(t)$$

$$0 = n_h^* n_h(t) - (1-\sigma_h) n_h^* n_h(t-1) - m_h^* m_h(t) - \eta n_l^* n_l(t-1)$$

$$0 = n_h^* n_h(t) + n_l^* n_l(t) + u_h^* u_h(t) + u_l^* u_l(t)$$

$$0 = u_h^* u_h(t) - (1-\gamma) u_h^* u_h(t-1) - \sigma_h n_h^* n_h(t-1) + m_h^* m_h(t)$$

$$y(t) - a(t) - \theta k(t-1) - (1-\theta) l(t)) = 0$$

$$l^* l(t) = n_h^* h_h^* [n_h(t) h_h(t)] + \tau n_l^* h_l^* [n_l(t) h_l(t)]$$

$$m_h(t) - \alpha_h v_h(t) - (1-\alpha_h) u_h(t) = 0$$

$$m_l(t) - \alpha_l v_l(t) - (1-\alpha_l) u_l(t) = 0$$

$$
\begin{aligned}
0 &= p_h(1-\theta)\frac{y^*}{t_h{}^*}h_h{}^*\left(y(t)-t(t)-h_h(t)\right)+p_h a_h\frac{v_h^*}{u_h^*}\left(v_h(t)-u_h(t-1)\right)+(1-p_h)b_h^*b_h(t) \\
&\quad +p_h\gamma\frac{a_h v_h^*}{m_h^*}\left(v_h(t)-m_h(t)\right)-p_h\gamma\frac{a_l v_l^*}{m_l^*}\left(v_l(t)-m_l(t)\right)+(1-p_h)\phi_{2,h}c^*\frac{(1-e)^{(1-\eta_h)}}{(1-\eta_h)}c(t) \\
&\quad -(1-p_h)\phi_{1,h}c^*\frac{(f_{h,t}^*)^{(1-\eta)}}{(1-\eta)}(c(t)+(1-\eta)f_h(t))-w_h^*h_h^*(w_h(t)+h_h(t))
\end{aligned}
$$

$$
\begin{aligned}
0 &= p_l(1-\theta)\frac{y^*}{l_l{}^*}\tau h_l^*\left(y(t)-l(t)+h_l(t)\right)+p_l\frac{a_l v_l^*}{u_l^*}\left(v_l(t)-u_l(t-1)\right)+(1-p_l)b_l^*b_l(t) \\
&\quad +p_l\eta\frac{a_h v_h^*}{m_h^*}\left(v_h(t)-m_h(t)\right)-p_l\eta\frac{a_l v_l^*}{m_l^*}\left(v_l(t)-m_l(t)\right)+(1-p_l)\phi_{2,l}c^*\frac{(1-e)^{(1-\eta_l)}}{(1-\eta_l)}c(t)- \\
&\quad -(1-p_h)\phi_{1,l}c^*\frac{(f_{l,t}^*)^{(1-\eta_l)}}{(1-\eta_l)}(c(t)+(1-\eta)f_l(t))-w_l^*h_l^*(w_l(t)+h_l(t))
\end{aligned}
$$

$$0 = u_h^*b_h^*(u_h(t-1)+b_h(t))+u_l^*b_l^*(u_l(t-1)+b_l(t))-\Upsilon_t\Upsilon(t)$$

$$b_h(t)-w_h(t-1)=0$$

$$b_l(t)-w_l(t-1)=0$$

B.2 Expectational equations

$$0=E_t\left[c(t)-c(t+1)+\beta\theta\frac{y^*}{k^*}(y(t+1)-k(t+1))\right]$$

$$0=E_t\left[\begin{array}{c}\frac{a_h V_h^*}{\beta M_h^*}(c(t)-c(t+1))+\theta_h\frac{Y^*}{N_h{}^*}h_h^*(y(t+1)-n_h(t)+h_h(t+1))\\ -w_h^*h_h^*(w_h(t+1)+h_h(t+1))+(1-\sigma_h)\frac{a_h V_h^*}{M_h^*}(v_h(t+1)-m_h(t+1))\end{array}\right]-\frac{a_h V_h^*}{\beta M_h^*}(v_h(t)-m_h(t))$$

$$0=E_t\left[\begin{array}{c}\frac{a_l V_l^*}{\beta M_l^*}(c(t)-c(t+1))+\theta_l\frac{Y^*}{N_l{}^*}\tau h_l(y(t+1)-n_l(t)+h_l(t+1))+\eta\frac{a_h V_h^*}{M_h^*}(v_h(t+1)-m_h(t+1))\\ -w_l^*h_l^*(w_l(t+1)+h_l(t+1))+(1-\sigma_h-\eta)\frac{a_l V_l^*}{M_l^*}(v_l(t+1)-m_l(t+1))\end{array}\right]-\frac{a_l V_l^*}{\beta M_l^*}(v_l(t)-m_l(t))$$

C Appendix: Macro-data

The macro-data used in this study is real aggregate data of the United States for the period 1964:Q1-2005:Q4. The source is the Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of Saint Louis.

- vacancies = help wanted advertising in newspapers / (population+16)
- employment = (civilian employment +16) / (population+16)
- unemployment = 1 - employment
- tightness = vacancies/unemployment
- total hours = employment*average weekly hours / (population+16)
- labor productivity = output / total hours
- Consumption: (real consumption of non durables + real consumption of services + government expenditures)/(population +16)

- Investment = (real consumption of durable goods+ real fixed private investment)/(population +16)
- output = consumption + investment + vacancies*cost per vacancy
- Normalization of output
  - output =1
  - consumption = Consumption / output
  - investment = Investment / output